

# AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

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## TRANSACTIONS.

NOTE.—This Society is not responsible as a body, for the facts and opinions advanced in any of its publications.

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### CCXLIII.

(Vol. XI.—August, 1882.)

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## HIGHWAY BRIDGES.

By JAMES OWEN, Member A. S. C. E.

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PRESENTED AT THE WASHINGTON CONVENTION, MAY 19TH, 1882.

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WITH DISCUSSION BY ASHBEL WELCH, PRESIDENT A. S. C. E.

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While it is not proposed to offer anything of a theoretical character in this paper, it has occurred to the writer that probably in an experience of over ten years in designing and superintending the construction of between 400 and 500 highway bridges of all sizes, from 2 feet to 275 feet span, costing in all about \$750,000, and of all characters, there would be probably many points obtained by his experience of advantage to the profession at large and to those interested in these structures in particular.

The district cared for embraces a territory of about 150 square miles, known as Essex County, New Jersey, with a population of about 200,000,

divided socially into city (Newark), 130,000; suburban, 50,000; and rural, 20,000.

The topographical characters embrace marsh and ordinary undulating land on the east in the city, undulating land for the suburban, and a dividing ridge, somewhat precipitous, in the centre, separating the suburban from the rural. The county is bounded on the west by the Passaic River, about 150 feet wide, and also on the east by the same river, but it is there navigable, and of a width of 300 to 400 feet. The Morris canal also traverses the county, requiring a considerable number of bridges. Other than this the streams are generally small, requiring few bridges over 30 feet span. The total number in the whole county is about 1200. The materials for construction were found *in situ*: good sandstone (Newark, Belleville), in the eastern part, trap rock in the centre, and an abundance of boulders in the western part, and an adjoining market (New York) for ordinary building materials.

Good white oak also is found in the western part. It will thus be seen that the district, furnishing all the governing circumstances, both physically and socially, that require different treatment in the bridge line, with good material and fairly liberal appropriations, gave a fair field for good work.

The governing principle for the work was to erect, if possible, nothing of a temporary character, so as to reduce the repair account to a minimum, the question of interest not entering as a factor of cost.

The main character of the bridges erected was as follows :

For waterways under 4 feet : Stone box culverts ; circular brick sewers ; cast-iron pipes.

From 4 to 25 feet : Brick arches ; stone walls, iron beams, planking ; stone walls, iron beams, brick arches and paving.

Over 25 feet : Wrought iron trusses with iron floor girders, stringers, and a roadway of planking, and occasionally a roadway of brick arches and paving.

Of course there are many exceptions to the above, in shape of wooden bridges of small and large dimensions, but nothing of a greater span than 50 feet.

In deciding for small spans, the following points were considered :

If the district was very flat, or there was a liability for the culvert to fill up by a sudden heavy shower, ordinary box culverts were used, built of rubble walls and bluestone covering. These, if filled up, could be

easily uncovered, the material removed and the covering relaid at small expense.

If there was a steady flow of water and no wash, cast-iron pipes—known as rejected—were very serviceable, and can be laid with a minimum of covering. If an area wider than 24 inches were required, brick sewers were used, but unless they are deep in the ground, backing is required, necessitating an increase of expense, and there is also a tendency for the water to wash out the mortar joints in the invert before they can set properly, causing the bricks to become loose and get displaced by a heavy freshet. The writer once constructed, for a mountain stream running through a village, a brick sewer 500 feet long, 6 feet in diameter, of 12 in. brickwork, without any backing, and with only 1 foot of covering over top of arch. The ground it was constructed in was good gravel, and everything stayed in its place, great care being taken to ram the gravel behind and on top. In any other soil this would be very risky. The only trouble was the washing out of the invert joints. This was obviated by creating a series of small dams, and then grouting. By this means the necessary tightness was attained.

Vitrified pipes are objectionable, on account of their liability to become choked, and are difficult to clean out; also to become broken, and latterly have been rarely used by the writer, except for spring brooks deep down. Cement pipes, on account of their fickleness, never.

In spans from 4 to 25 feet, where there is plenty of height between level of brook and highway, an arch bridge is by far the best, but, except in a purely agricultural region, there are objections to raising roads, any more than is absolutely necessary, above the normal grade of the ground. So flat bridges were mostly built of stone walls, with either wooden beams (these rarely) and planking, iron beams and planking, or iron beams, brick arches and paving—the latter by far the best, there being no necessity for repairs.

For bridges above 25 feet, trusses of wood or iron become necessary, iron being the rule, and wood the exception.

In deciding on the area for water way, the following general rules were adopted :

It might be stated here that the writer found that any calculations for the required area based on any formula that he could find, were utterly inadequate in their results; for he found that the

action of the storms in the hilly regions was very capricious. A sudden heavy fall of rain in a very limited area would create a flow of water that only experience could appreciate, and he was therefore compelled to rely on experience and the knowledge of inhabitants in the neighborhood as a guidance.

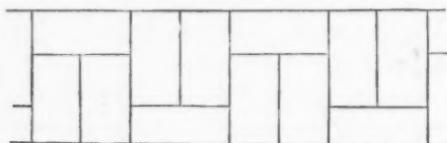
But as the region was an old-settled country, this experience was of practical value, and of all the bridges built within the last ten years, only four have suffered seriously by freshets.

If a new bridge were built on the site of an old one, the area was increased from 30 to 50 per cent., and in some cases doubled; if on a new site, comparisons were made from the nearest bridges, and the area always increased 20 to 50 per cent., it being very necessary to remember that in a country increasing in population, an increased area of water way is required, on account of the many improvements made. Every house built, every street graded, causes the water to be shed much more quickly, and bridges should be built to the extreme, not the average, amount of water.

The writer has also been led to condemn the practice of dividing the water way of bridges less than 50 feet into a number of spans. The necessary piers are a great detriment to the flow of water, intercept brush and logs, causing, in some cases, a complete obstruction, and are liable to become undermined. The saving in the first cost is small in comparison to the probable resulting expenditure afterwards.

The following details in construction may be of service :

*Foundations.*—For all foundations less than 4 feet in thickness, the specifications were made to require that the excavation be carried to at least 2 feet below level of bed of stream, and that all foundation stones shall be at least as long as the width of the abutment, 2 feet wide and 1 foot thick. This is a good rule to adopt, as it admits of no dubious work. If the abutments are wider than 4 feet, the stones were specified to be laid in this manner, which makes good work :



If quicksand is struck, the rule is to excavate to the full width of the bridge, and to lay 2-inch planking longitudinally under the site of the abutments, and to spike to them, crossways, planking as long as the sum of the width of the opening, and the two abutments. This is not costly, is absolutely secure, the cross planking preventing any scouring during a freshet, it being 2 feet below the normal bed of the stream. This mode is available up to 25 feet span ; over that it would not be advisable to lay the planking all the way across, but merely underneath each foundation.

In building foundations in the marshes, piles were driven, the specification requiring a limiting penetration of  $\frac{1}{4}$  inches with a 1200 pound hammer falling 20 feet. Hard bottom was always reached, but the upper stratum being soft mud, the caps were braced across the opening, as there was a tendency for the mud to be forced up from under the abutments into the channel by the filling behind, which brought a cross strain on the tops of the piles.

*Masonry*.—There is nothing to note specially about masonry, except that the writer has found it better to specify (except in small culverts) first class or coursed masonry, instead of rubble work, as he found he could get good coursed masonry when he couldn't get good rubble work, even from the same men. Of course the price was higher, but there were no big joints to wash or freeze out, so no repairs afterwards.

The main points required were good stone, full beds and plenty of headers, limited to 5 feet in their distance apart on each course, thus obviating any veneering, the great curse of ordinary masonry.

*Arches*.—With very few exceptions, the arches (except their ends, which were of dressed stone) were all turned of brick, as a stone arch is not equal to a brick arch, unless every stone is dressed to a radius, which is more costly than brick, and a stone arch of ordinary flat or field stone (many of which exist) is a treacherous structure, and sooner or later requires renewing.

The arches should have solid skewbacks dressed to the radius, the bricks laid in rowlocks keeping each course distinct ; the centre row on top driven snugly in a bed of mortar, and the centres struck as soon as the arch is turned. This allows everything to settle to an even bed. The practice of driving wedges of brick to key the arches, and then grouting, is objectionable, for where the wedges are the grouting does not flow, and the wedges take all the strain.

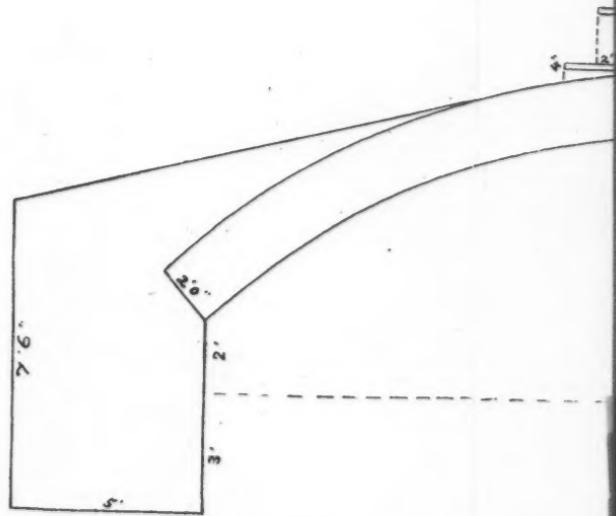
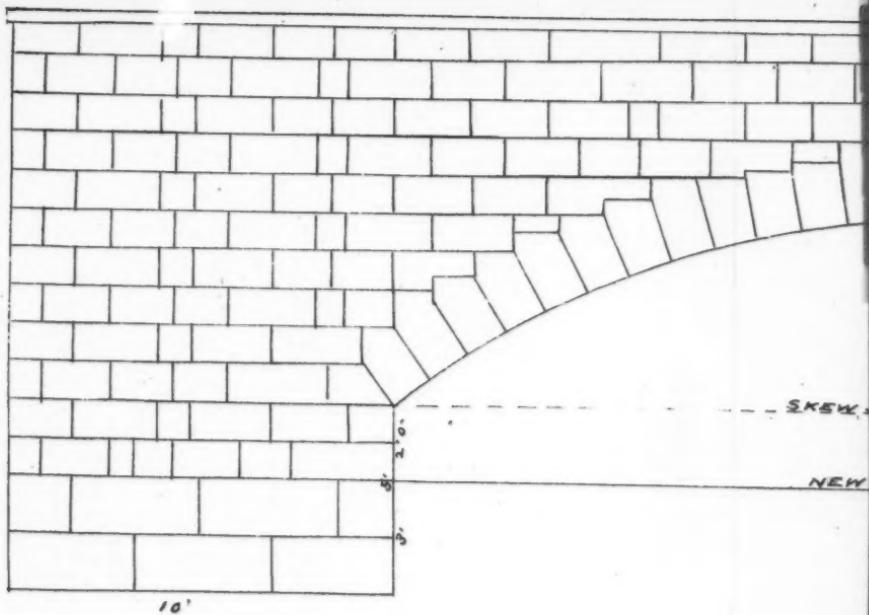
The settlement of an arch 28 feet span, 5 feet rise and 20 inches thick was found by the writer to be exactly  $\frac{1}{2}$  inch. In skew arches, as the length as a rule was much more than the span, the spiral courses were only laid sufficiently far from the face to give a firm rest on the shortest abutment; the centre work being laid square, a butting face, not bonded, being made between the two. The writer has built them this way of  $45^{\circ}$  and 25 feet span, and they are very satisfactory. (See Plate XXVIII.)

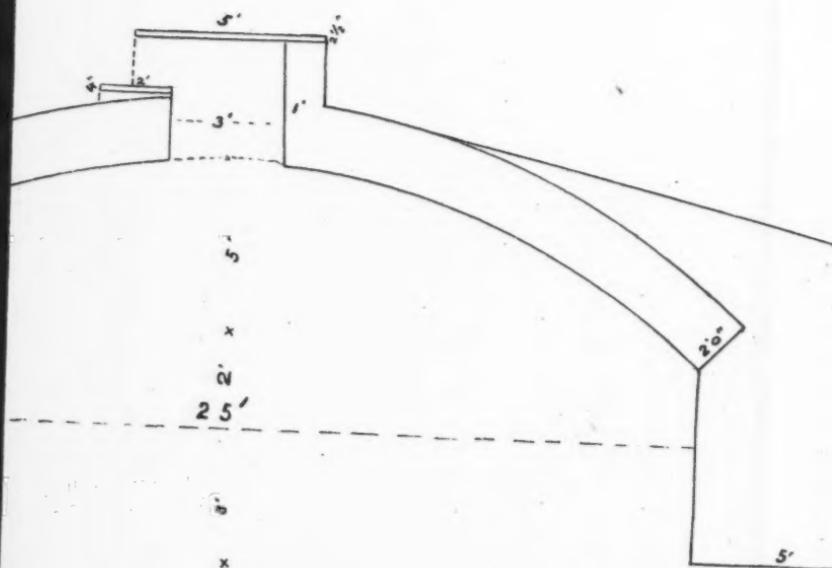
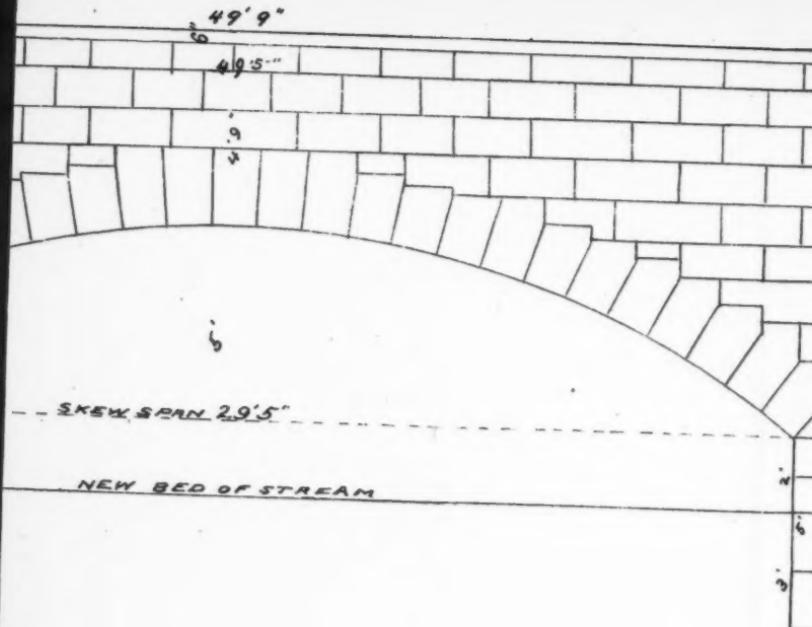
As almost all the bridges were over streams, the question of preserving intact the water way became very important, so a rule was adopted to put a paved invert in all water ways, except where planking existed, for though in places the stream will remain normal or even fill up, yet there is an uncertainty of the former, and in the latter the paving somewhat prevented the silting up, and when clearing by hand was found necessary, it preserved a limit, and gave a guide to the necessary digging.

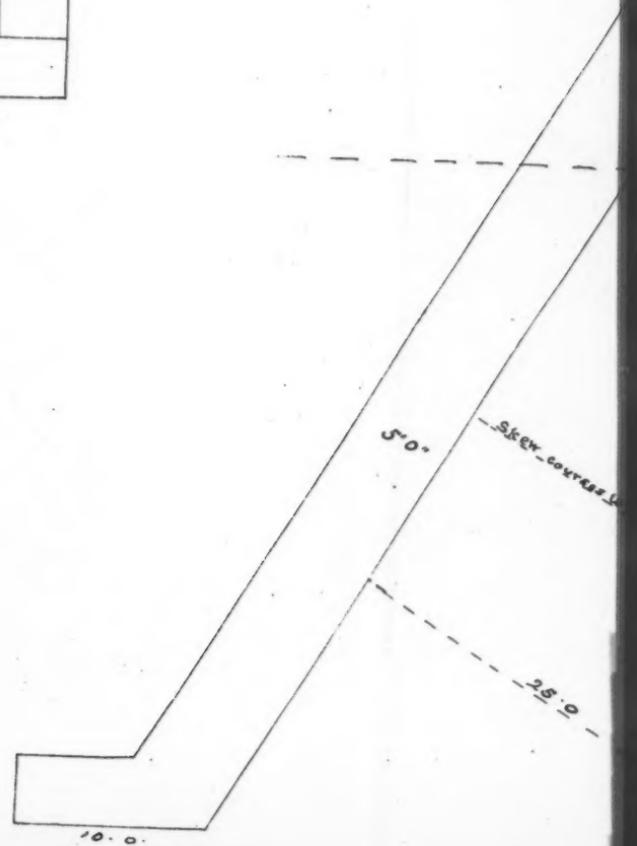
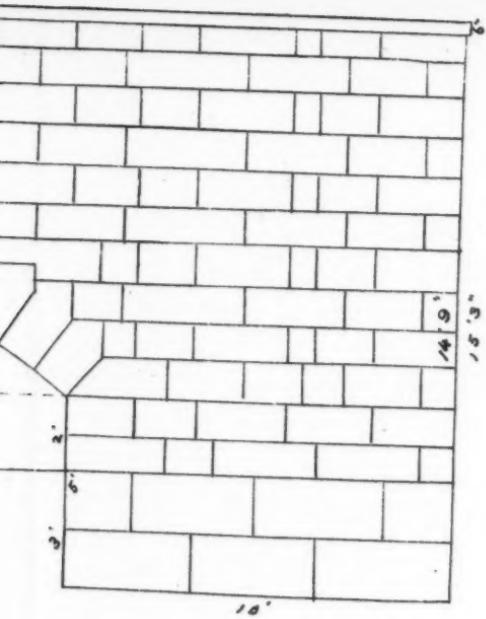
The paved invert were laid extending 10 feet beyond each end of bridge, with stones 12 to 24 inches in diameter, their longest end downwards, to a proper distance in gravel. If the current is extremely rapid at times, or if the grade is steep, timber curbs 10" x 12" were laid at each end of paving, with their ends properly anchored in the ground. If, as sometime occurs, the length is too much to properly anchor at both ends, insert anchor post 3 feet deep in bed of stream, and bolt the curb to this. Another way is to run the lower end of the paving into the bed of the brook at a steeper grade than that of the brook itself. In some cases the writer has found it necessary to build walls 6 to 10 feet high at end of paving, the water in a few years having eroded to that extent.

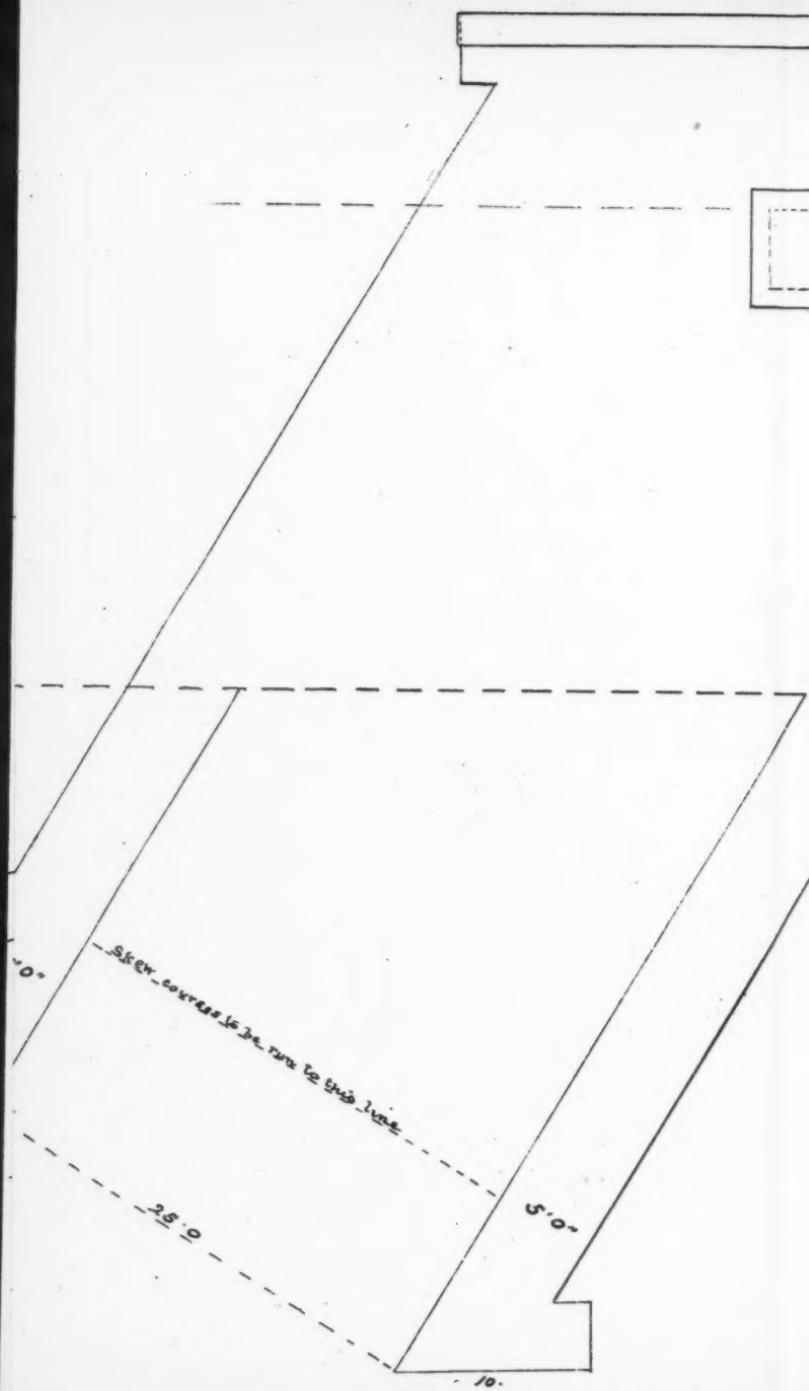
Where the water way has for some reason or other been cramped, the paving under the bridge has been laid two feet below the general level of the bed of brook. This, in ordinary times, becomes silted up, but when a great rush of water occurs and the opening is filled, a head is created, the silt is scoured out, and the bridge acts as an inverted syphon, and more water way is created. Care should be taken that abutments are deep enough, and the bridge has sufficient dead weight to resist any lifting tendency of the water. This answers the purpose admirably, and when tried has been successful.

*Timber Bridges.*—As the writer has stated that but few wooden bridges have been built, he has but little to say on that head.









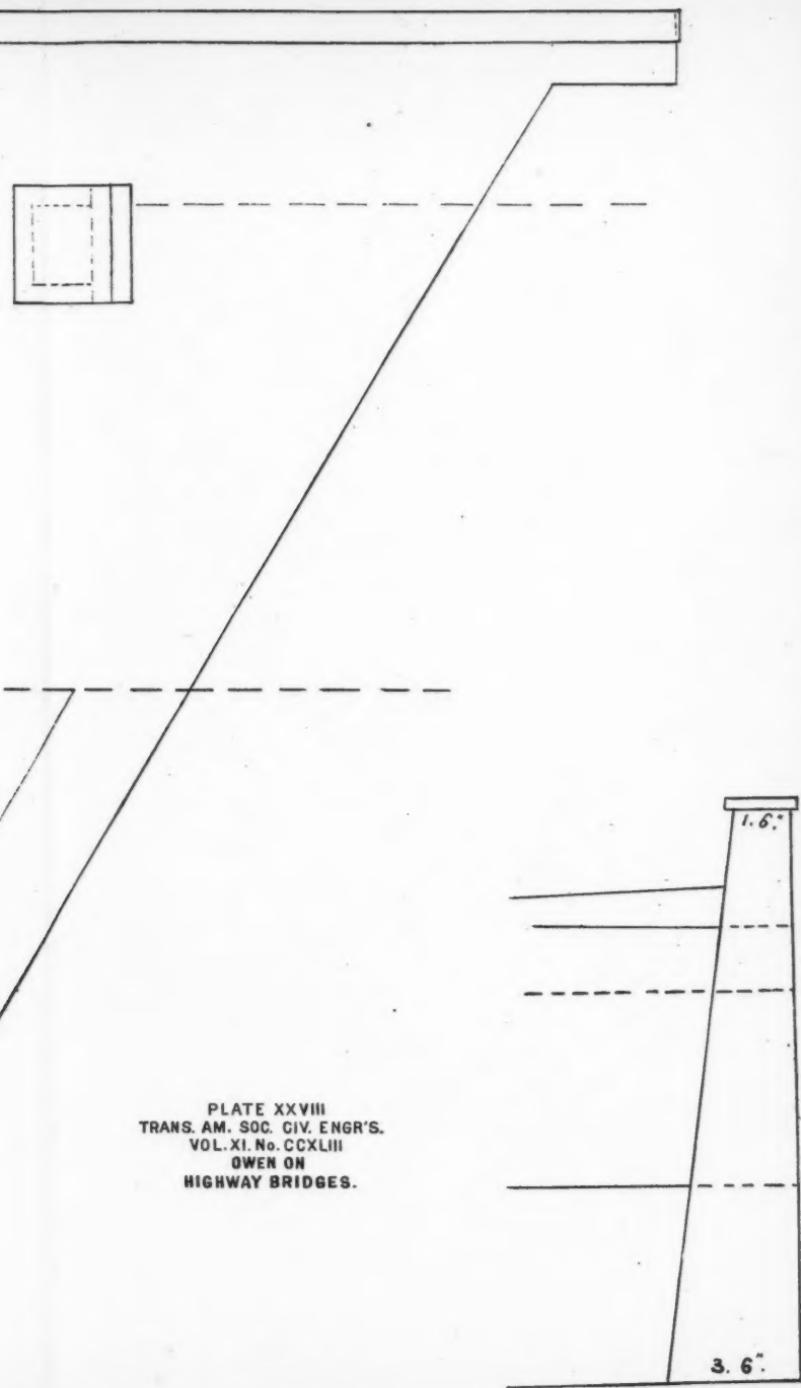


PLATE XXVIII  
TRANS. AM. SOC. CIV. ENGR'S.  
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OWEN ON  
HIGHWAY BRIDGES.



For spans under 20 feet the writer found it best to put in wooden beams, the size in proportion to span, 2 feet apart, well bridged with solid blocks, herring bone bridging being objectionable on account of the tendency it has to work loose.

The most satisfactory arrangements at end of beams for the planking is as per drawing. (See Fig. 1, Plate XXIX.)

For spans over 20 and under 30 feet ordinary A trusses were used, with no special feature, except that in some cases it was found necessary to put in white oak braces instead of a softer wood, which became rapidly whittled away by the unemployed inhabitants of the neighborhood.

Mixing sand with the paint prevents, in a degree, the whittling.

For spans of 40 to 50 feet, queen trusses were used. It is an old type, but seems to be the best, allowing but little vibration. The needle beams project for the outside braces, which consist of a 6-inch by 4-inch timber, and a 1-inch rod, with allowance for adjustment; on tightening this, the truss was very rigid and satisfactory.

In proportioning wooden trusses and beams, the writer made allowance for future decay.

A very good arrangement to prevent the floor beams from decay was to cover the tops with ordinary tarred paper, with a lap on each side to let the water drip off. There are yellow pine beams covered in this way, put in 17 years ago, doing good service.

In framing trusses, specifications required that all joints be coated with white lead before being put together, and that all spaces between iron and wood be filled with the same material.

*Iron beams and planking.*—This makes a fairly satisfactory bridge for the rural districts.

Ordinary rolled beams were used up to 20 feet span, the sizes and distances apart varying as the span—the limit of 4 feet being made in the latter. The writer has as yet, discovered no satisfactory way of fastening the planking to the beams. The usual way of bolting a strip to the iron beams he has discarded, and inserts wooden beams, the same depth as the iron beams—masons in the ends—3 to each plank, and spikes the planking to them; he acknowledges it is cumbrous and is open to suggestion.

The question of planking for roadway, is one of considerable importance as it is a great element in the repair item. The following is the practice adopted:

Where planking, as in the rural districts, will rot out before it is worn out, Jersey white oak, which is an exceptionally hard, durable timber, is used; in cities under the same conditions yellow pine, it being there less costly; in cities where it will wear out before it rots out, spruce. This wood stands the wear well. The usual thickness is 3 inches, sometimes 4, but in all cases the following is specified: Lay no plank wider than 9 inches. This prevents wide joints in shrinkage. Bore all holes for the spikes, to prevent splitting, and put no spike nearer than 4 inches to the end of planking; this necessitates a double row of spiking beams where joints come, but it is the best way of obviating the nuisance of loose ends.

*Iron beams—Brick arches.*—This, in the writer's opinion, is the standard bridge for highways, for when done it is a finished structure. (See Fig. 2, Plate XXIX, and also Plate XXX.)

Rolled beams were used up to 20 feet span, beyond that plate girders were necessary, tied together with  $\frac{1}{4}$ -inch rods, parallel with abutments 4 feet apart.

Brick arches were then turned between the beams or girders, ordinarily with  $\frac{1}{6}$  rise; where the spaces between the beams were less than 2 feet 6 inches, 4-inch arches were put in; beyond that, 8 inches up to 7 feet span, the limit. The arches were then levelled with concrete and smoothed off with a good coat of mortar, then a water proof covering of tarred paper laid in a mixture of tar and asphalt, on this 2 inches of sand, and then the broken stone or paving blocks.

In case where the sidewalk was higher than the roadway, the beams were built in a little higher, on these flagging was laid with pitched edges bolted down and joints leaded, the raising of the sidewalk leaves a convenient opening for gutter water to flow away. (See Plate XXX.)

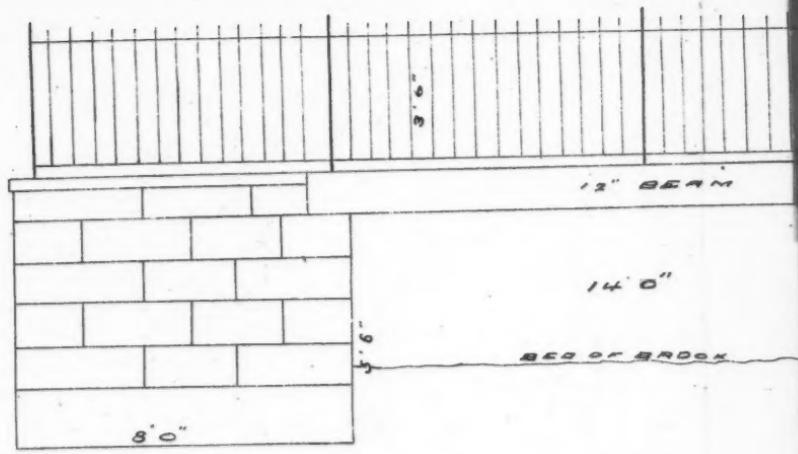
If broken stone was used, the thickness was 12 inches in centre tapering to 6.

The beams carrying so much dead load were calculated with a factor of safety of 3.

*Railing.*—Probably no other detail has been so troublesome to handle as the railings for the bridges. Theoretically they would seem simple enough, but practically there is great difficulty in getting them permanent.

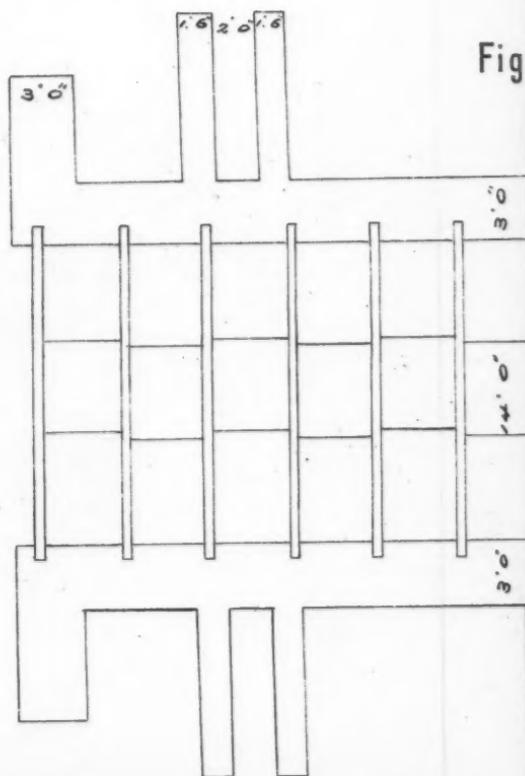
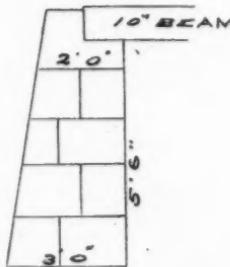
It seems to be one of the inherent traits of humanity to employ its idle hours sitting, if it can, on the railing of a bridge; another trait is,

BRIDGE FOREST AVE.

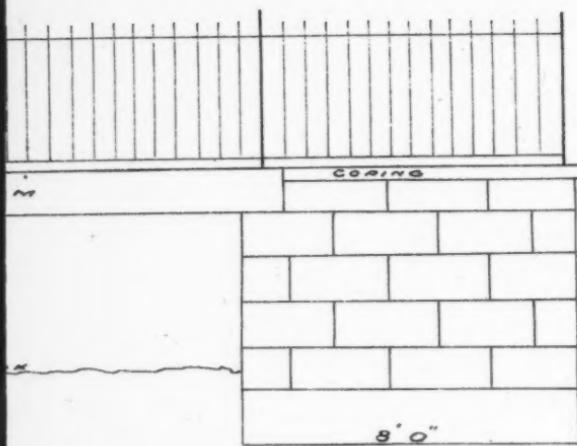


Fig

SECTION  
OF  
WALL



VE. NEWARK



72" BEAM

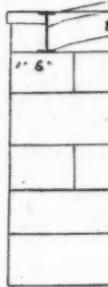
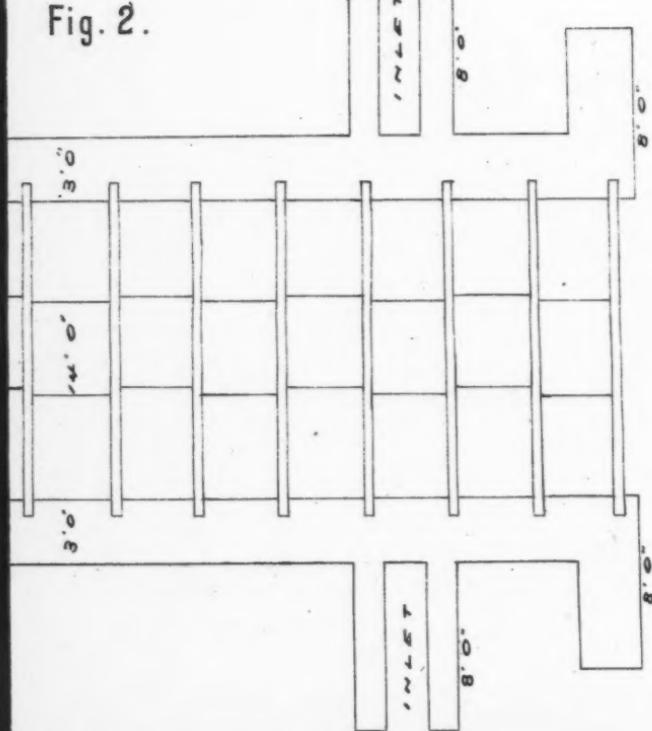


Fig. 2.



SECTION SHOWING  
ARCHES

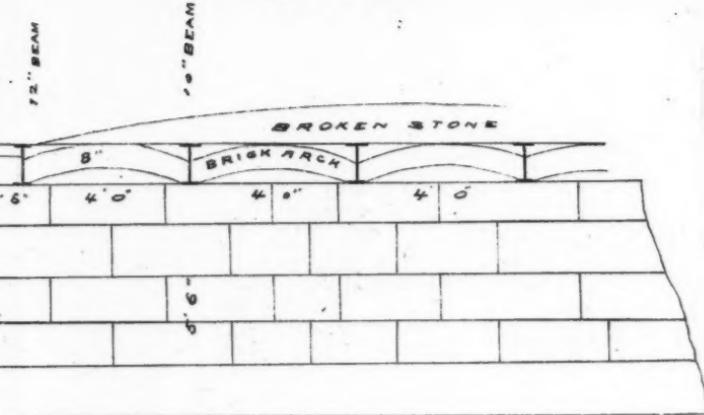


PLATE XXIX  
TRANS. AM. SOC. CIV. ENGR'S.  
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OWEN ON HIGHWAY BRIDGES.

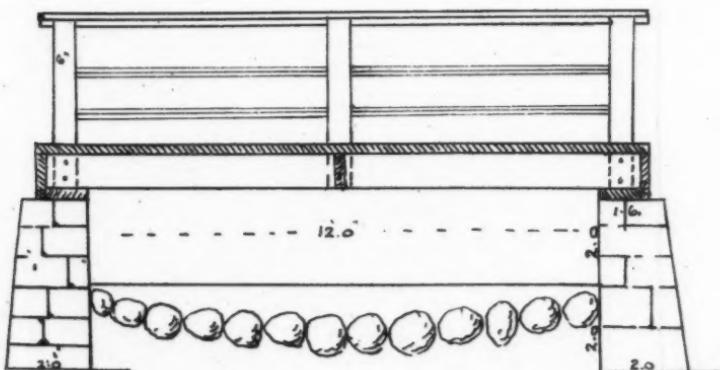
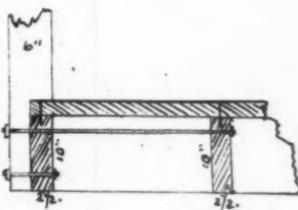


Fig 1

WOODEN BRIDGE





if the railing is of wood, to whittle it all away with praiseworthy energy, or if there is any give or spring to it to experimentalize as to the limit of the give or spring; if there is an opening large enough for a baby to creep through, the baby is always on hand to fulfil its seeming mission. None of these points are even hinted at in the text books on engineering, but they are potent factors in the railing question.

In trying to overcome these difficulties the writer has adopted two styles of railing as standard, one of wood and one of iron, which he adheres to as much as he can.

The wooden railing consists of oak, which is difficult to cut, the posts 6 inches square, 3 rails, the top 4 inches square, the two lower 3 inches square, set corners upward, so as to be uncomfortable to sit on. The attachments are shown on the drawing. (Plate XXIX.)

This makes a good railing for rural districts and small streams.

The iron railing is made of wrought-iron pickets 1-inch square, 6 inches pitch, varying from 3 feet 6 inches to 5 feet high, with blunt points, this should be braced carefully, is neat in appearance and strong.

All the fancy designs submitted are a snare. If of cast-iron, a small boy with a base ball bat soon demolishes them, and if of wrought iron, they generally are too frail or too expensive.

This probably exhausts all the remarks to be made on the subject, except for wrought-iron truss bridges, and this branch of the work has been so extensively treated by so many talented engineers, it would be useless for the writer to say much about them, except to allude to two things :

The wrought-iron highway bridge work is an extensive and increasing industry in the United States, and has practically become a business of itself. Workshops are now kept employed with this work and nothing else. To this there can be no objection, provided the business is carried on in a manner that shall in no way be open to criticism. There are hundreds of bridges built in this country of long and short spans, with no other assurance of strength than the guarantee of their builders ; if this guarantee were a professional one, nothing more could probably be asked, but in very many instances this is not the case. The company or individual who builds the bridge builds it to make money. Competition and inherent avidity cause a tendency to cut down material, and light and insecure bridges are too often the result.

The writer only alludes to this matter generally, for as far as his

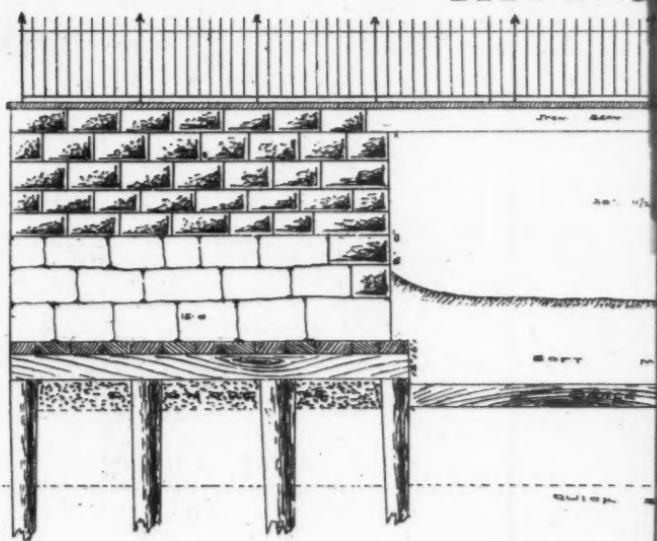
experience goes he knows but little. The rule he adopted was to draw up a general specification defining loads and limiting strains; then only calling upon iron bridge builders of unquestionable repute to bid was, in his opinion, the only satisfactory way of attaining good work—he considering the question of life and death a serious matter, and with which no risk should be incurred or even the question of risk raised.

The other point is one which seems, as yet, to have had but little attention given to it, viz., lateral and vertical vibrations. The writer offers no theories in the matter, but, in his opinion, all highway bridges, up to 75 feet, or even 100 feet span should be riveted structures, as that form of construction according to his observation seems best able to absorb or prevent this vibration. There are pin-connected bridges in his jurisdiction that have carried a 20-ton steam roller (giving a load of over 300 pounds per square foot) with little deflection, which will quiver and vibrate to a great degree at the passage of a fast pony and carriage, the tension members in the trusses will so rattle that they have to be bound with wire, the nuts, unless the heads of pins are thoroughly upset, will work loose and have to be replaced, and the floor bracing will undulate. This condition of things seems to the writer very objectionable, and though no defined damage takes place, it is very probable that the molecular condition of the iron will more quickly be changed, and its strength deteriorated.

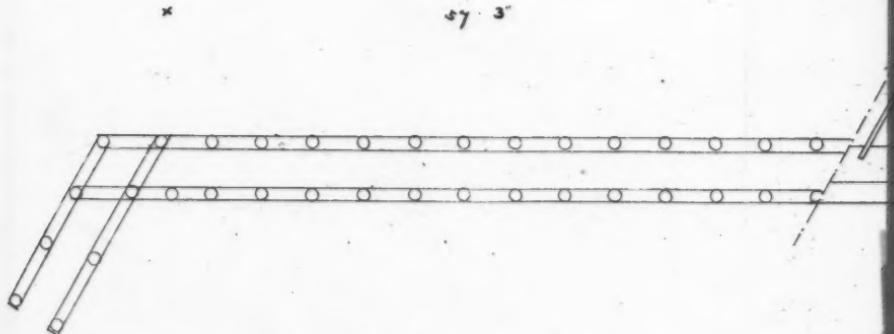
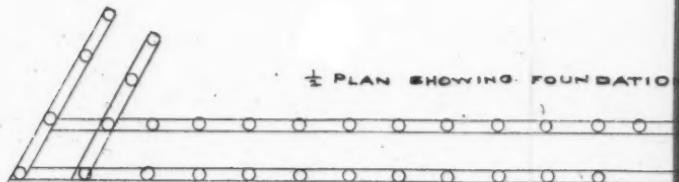
To obviate this as much as possible, the writer, for the last few years, has given preference to riveted trusses of low depth with angle iron bracing and triangular panels; a structure of this kind has few flat or round bars to vibrate, and is to a certain extent homogeneous; he alludes to this practice merely as a suggestion, and is open to correction if this or any of his other practices should be in any way wrong.

In conclusion, he would allude to one fact in connection with highway bridges, viz., that in the present status of the laws in the United States, the control of them must be vested in officers elected by the people or appointed, and is to an extent political. It is so in the writer's case, but he wishes here to have it put on record that he was always seconded in his efforts to keep his bridges out of politics, always seconded in his efforts to have good work done, and always seconded in his efforts in erecting structures that should give a fair equivalent for the money spent; more, in his opinion, cannot be asked.

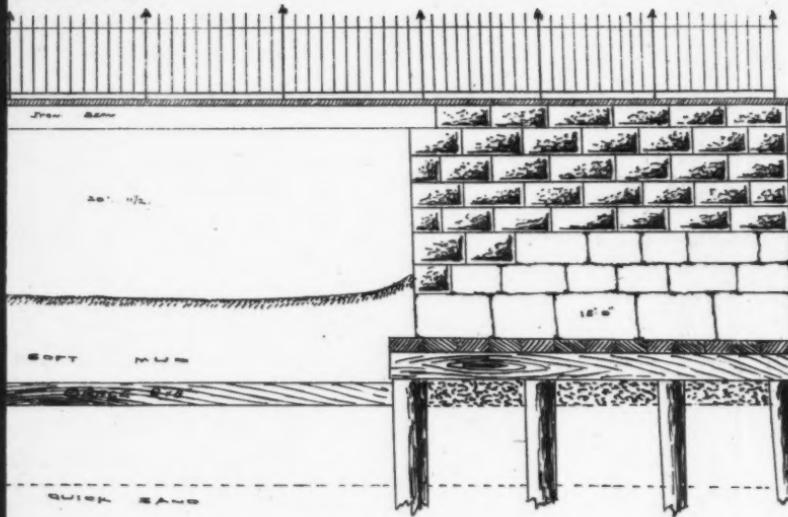
BRIDGE ON FRELINGHUYSEN AVE NEWARK  
ELEVATION



PLAN SHOWING FOUNDATION

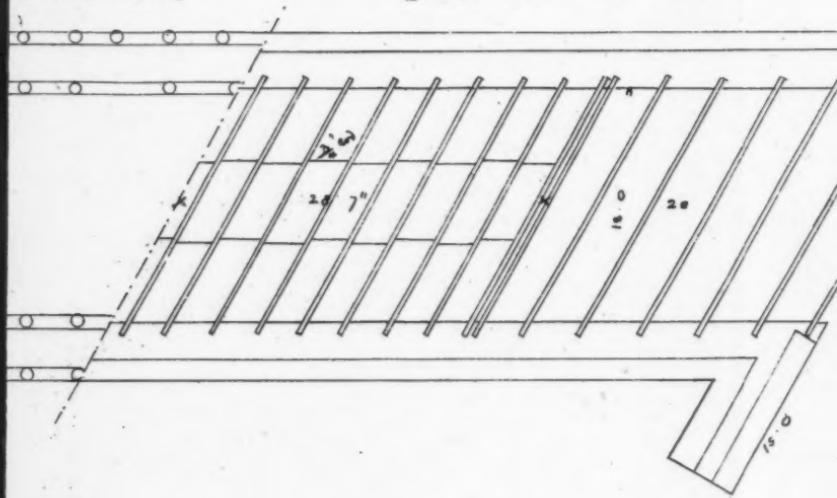


NEWARK  
VATION



FOUNDATIONS

PLAN SHOWING BEAMS



LONGITUDINAL SECTION OF ROADWAY

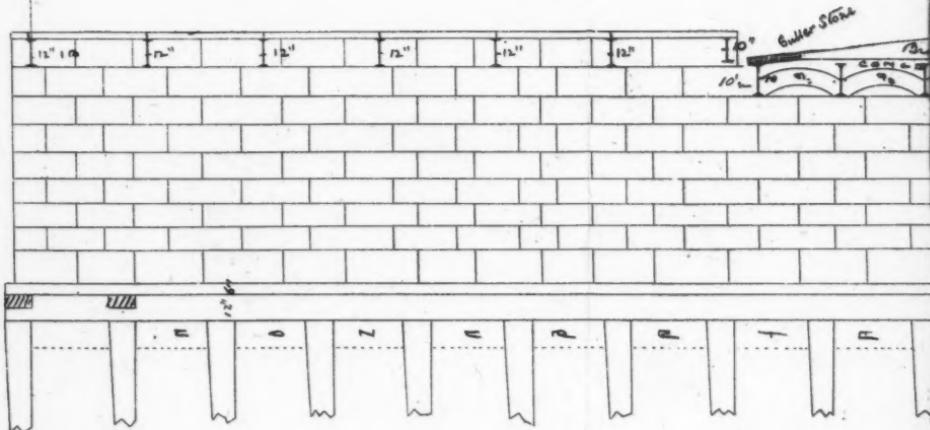
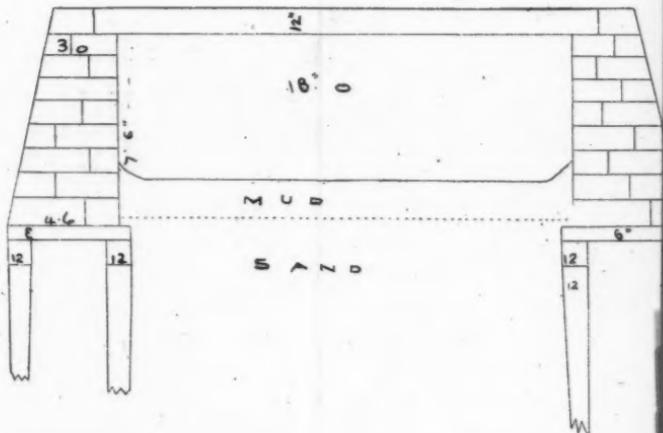
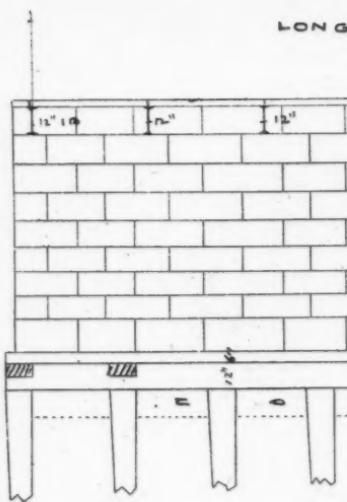
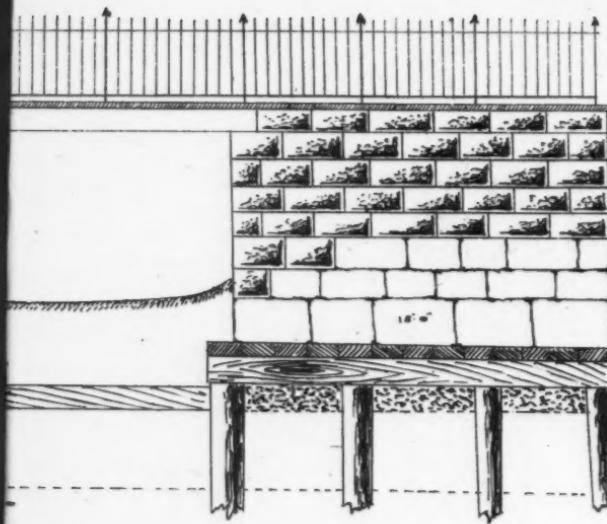


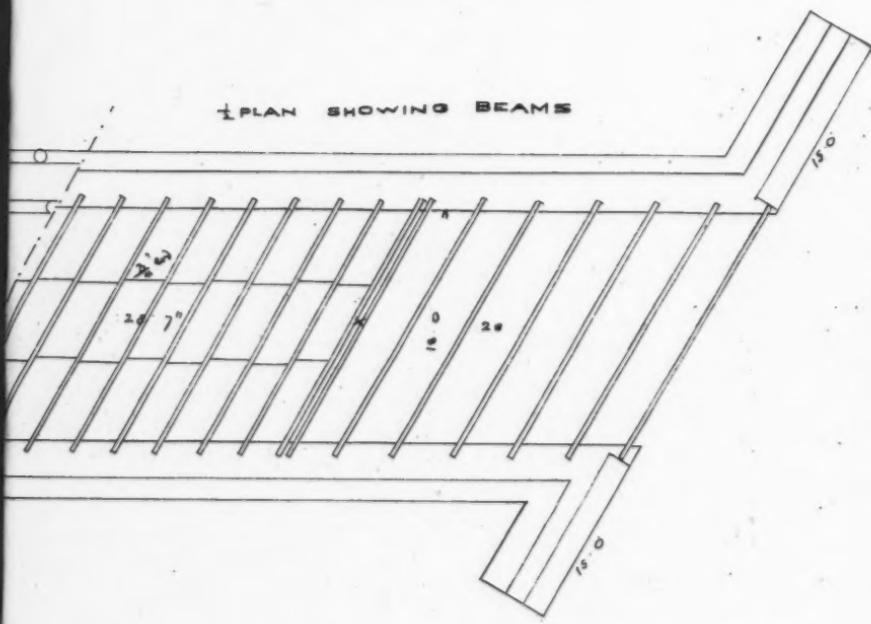
PLATE XXX  
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HIGHWAY BRIDGE

CROSS SECTION





PLAN SHOWING BEAMS



LONGITUDINAL SECTION OF ROADWAY

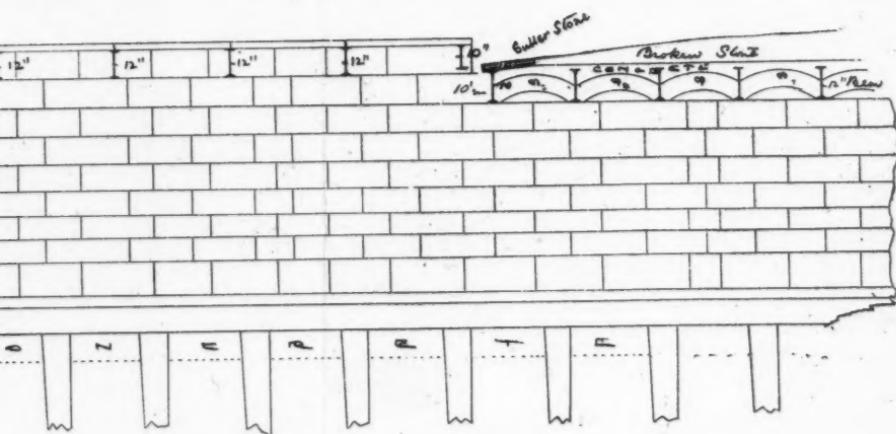
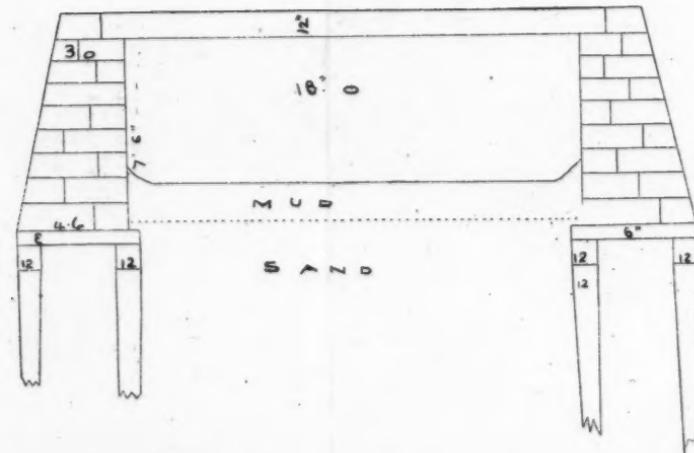


PLATE XXX.  
TRANS. AM. SOC. CIV. ENGR'S.  
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HIGHWAY BRIDGES.

## CROSS SECTION



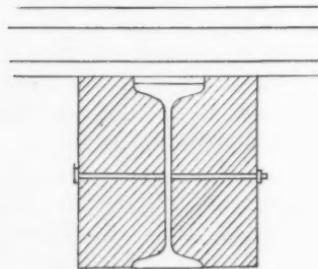


There are presented herewith Plates XXVIII, XXIX and XXX, showing a Stone Arch Skew Bridge, a Wooden Bridge, an Iron Beam and Brick Arch Bridge, and an Iron Beam Bridge with Brick Arches under Roadway and Flagging under Sidewalk.

DISCUSSION BY ASHBEL WELCH, PRESIDENT A. S. C. E.

In regard to the amount of water flowing away from particular storms, I know of an instance where a stream draining not over three square miles, filled, towards the end of a very hard rain, a culvert of 170 square feet sectional area, and at the upper end rose two or three feet above the top of the arch, so that the fall through the culvert was perhaps four or five feet, and the velocity at the lower end sixteen or twenty feet per second. The basin of this stream is very hilly and the rock, often bare, an indurated red shale. Probably 90 per cent. of the water ran off within less than an hour after it fell. But this is a very extreme case.

In a railroad bridge where was necessary to economize height, I suspended iron cross beams 12 feet deep below the truss, and bolted an oak scantling, carefully fitted, on each side, rising half an inch above the iron, and spiked down the rail directly on the scantling, thus :—



So that the distance from the top of the rail to the bottom of the beam was 17 inches. Mr. Owen has adopted something like the same plan in road bridges, but has not found it necessary there to bolt the timber to the iron.

When the foundation of a culvert or bridge of small span is sure to be always wet, I have always found it the cheapest and far the safest to use a timber platform, well sheet-piled, and carried some little distance

above and below the arch or straight bridge walls. The bed timbers may be of flattened hemlock or other cheap timber, a foot more or less deep, according to span and soil, running across the opening and a little beyond the outside of each abutment, covered with 2 or 3 inch plank according to circumstances, thus :



I never knew but one such foundation to give way, and that was very badly constructed, the foundation timbers not running across. On the contrary, I have known many bridges and culverts to give way when the abutments were sunken, and the span between well paved. A few years ago a culvert at Milford, N. J., of two semi-circular arches of 25 feet span, the abutments of which were sunk several feet below the pavement, and the pavement, two feet thick, was extended some 50 feet down stream from the lower end of the culvert, and well sheet-piled, gave way from undermining, first below the lower end of the pavement and then working up. A wooden platform would not have given away. Mr. Owen has gained much in safety by the wooden apron he places at the lower end of his pavement.

In one case when a violent rain flood had cut a hole 8 or ten feet deep at the lower end of a timber foundation of a culvert of 2 arches of 25 feet span, I sank a crib some 15 feet wide in the cavity across the whole width of the stream, and continued the floor of the foundation platform over it. The floods of 40 years since have not disturbed it.

In a neighboring county to that in which Mr. Owen has operated, they formerly used pin oak extensively for bridge floors. Pin oak plank  $2\frac{1}{2}$  inches thick, seasoned very hard all through, but if 3 inches thick the plank did not season through, but rotted in the middle of its thickness, and gave out much sooner than the thinner plank. Sweet gum, or *bilsted*, as we call it in New Jersey, wears indefinitely, but does not stand the weather. In the old freight transhipment house at South Amboy is a floor which was trucked over night and day for probably a quarter of a century, and which shows almost no marks of wear. White cedar, or one kind of cypress, wears even better. These soft, elastic, strong fibred woods, yield to blows or pressure but without breaking the

fibre, and resume their original form, while most hard woods are permanently indented by the same pressures or blows, and their fibres gradually broken.

The Trenton Delaware Bridge built in 1804 at a cost of \$180 000 (a vast sum in those days) and which for many years was the most famous structure of the kind in America, was supported on white pine arches 160 feet span, 12 inches wide and about 2 feet deep. The arches were made up of pine plank. In 1837 I placed a railroad track on this bridge, adding a few inches to the depth of the arches. Where the arch was completely protected from the weather I found the timber perfectly sound. But where the rains sometimes reached it, and where the joints between the planks that formed the arches were not close enough entirely to exclude the water, nor open enough to let it entirely dry out, the outsides of the arches for a thickness of 2 or 3 inches were of bony hardness, all the middle part of the wood for about half its whole thickness, was of the consistence of scotch snuff. The auger could be pushed 6 inches through it without turning. Of course all such parts of the arches were removed.

The question often arises whether a cheaper bridge that will last a shorter time is more economical than a more costly one that will last a longer time. The same question may arise respecting many other things.

To find the comparative economy of two things of different cost and durability, that will answer the same purpose equally while they last, the following formulae will be found convenient :

Let  $C$  be the cost and assumed real value of one of them,  $T$  the time it will last,  $a$  the compound interest on one dollar for that time, at whatever rate money is worth to the party using the thing or costs that party, and  $L$  the loss of the thing when done with, which may or may not be equal to  $C$ ; let  $R$  be the real value for the purpose of the other thing,  $C'$  its cost,  $T'$  its duration,  $a'$  the compound interest for that time, and  $L'$  the loss on it; and let  $V$  be the value of the thing for that purpose that would last forever if all circumstances remained constant.

$$\text{Then } V = C + \frac{L}{a}$$

$$R = \frac{a' V}{1+a'} \text{ that is } R = \left( C + \frac{L}{a} \right) a' \div (1+a')$$

The difference between  $R$  and  $C'$  is the advantage or the disadvantage of the thing whose cost is  $C'$ .

Suppose a bridge that will last seven years costs \$8 000, and the loss at the end is just the cost, and money costs the parties interested 7 per cent., what would be the equivalent value of a bridge that would last five years, and of one that would last forever?  $8\ 000 + \frac{8\ 000}{.62} = 20\ 900$ , the value for that place of the bridge that would last forever, and  $(.41 \times 20\ 900) \div (1 + .41) = 6\ 077$ , the value of one that would last five years.